RCC 266-16

Range Commanders Council

ELECTRONIC TRAJECTORY
MEASUREMENTS GROUP

SURVEY OF RADAR REFRACTION ERROR CORRECTIONS

ABERDEEN TEST CENTER
DUGWAY PROVING GROUND
REAGAN TEST SITE
REDSTONE TEST CENTER
WHITE SANDS MISSILE RANGE
YUMA PROVING GROUND

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NAVAL UNDERSEA WARFARE CENTER DIVISION, NEWPORT
PACIFIC MISSILE RANGE FACILITY

30TH SPACE WING
45TH SPACE WING
96TH TEST WING
412TH TEST WING
ARNOLD ENGINEERING DEVELOPMENT COMPLEX

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SURVEY OF RADAR REFRACTION ERROR CORRECTIONS

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Electronic Trajectory Measurements Group

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New Mexico 88002-5110

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Preface

This document introduces basic refraction error estimation for an electromagnetic wave propagating at radio frequencies through the earth's atmosphere. Appendices contain descriptive material on the process, algorithms, and application of refraction correction methods to radar tracking data at each of the participating ranges.

This document is a standalone document; however, it is intended as a prelude to the development of a standard for correcting of radio frequency refraction errors occurring in radar track measurements.

This document was prepared for the Range Commanders Council through the Electronic Trajectory Measurements Group (ETMG). For questions regarding this document, please contact the Range Commanders Council Secretariat office.

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Acronyms

BAE	BAE Systems
CRPL	Central Radio Propagation Laboratory
EM	electromagnetic
ETMG	Electronics Trajectory Measurement Group
FPS-16	Army/Navy designator for a class of instrumentation radars used at many test ranges
MATLAB	Registered trademark for mathematical software provided by MathWorks Inc.
ppm	parts per million
RCC	Range Commanders Council
RF	radio frequency
RIR	Base designator for a class of instrumentation radars developed by BAE Systems
WSMR	White Sands Missile Range

1. Introduction

As a prelude to the development of a standard for correcting of radio frequency (RF) refraction errors, the ETMG is surveying its member ranges to collect information on the processes and algorithms currently used for correcting these errors.

This short document introduces basic refraction error estimation. The appendices to the document contain descriptive material on the process, algorithms, and application of refraction correction methods to radar tracking data at each of the participating ranges.

2. Refraction

The speed of propagation of an electromagnetic (EM) wave is constant only in a vacuum. The speed of propagation through any other medium is dependent on its electrical properties.

The medium of interest is, of course, the earth's atmosphere; however, the earth's atmosphere is not a constant medium: its properties change with temperature, pressure, and humidity. Generally, the speed of propagation will vary continuously as the EM wave travels from the radar to the target and back again.

Refraction occurs as the changing properties of the atmosphere constantly alter the speed of propagation. For a radar located on or near the surface of the earth the wave is continuously bent downward. The effect is that the ray follows a curved rather than straight path through the atmosphere.[1] Figure 1 demonstrates the effect.

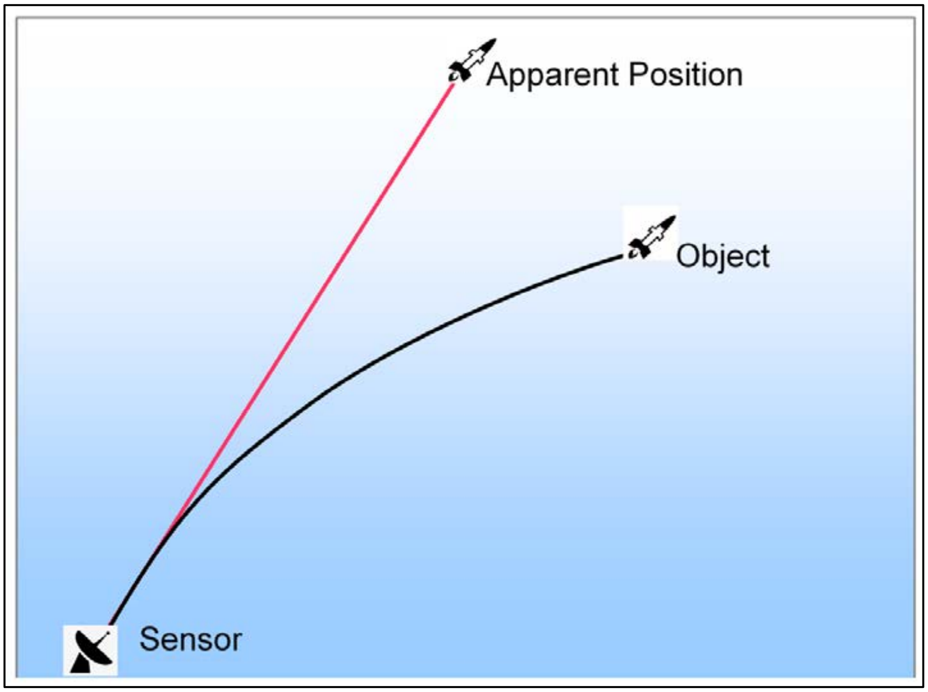

Figure 1. Refraction Effect

[1] Physical Science Laboratory. "Data Systems Manual, Meteorology and Timing." Prepared for White Sands Missile Range under contract DAAD07-76-0007, September, 1979. Pp31-53. Available on request to the RCC Secretariat.

Refraction changes the apparent altitude of the object so that it seems higher than it actually is. The apparent distance to the object is also affected because the curved path is slightly longer than a direct path to the target.

Refraction correction is an attempt to model the systematic behavior of the EM wave as it moves through the atmosphere and then correct its effect so that a more accurate measurement is obtained.

A starting point is to estimate the speed of the EM wave as it travels through a medium other than a vacuum. Basic EM theory[2] shows the speed of propagation through a medium depends on its electrical permittivity (ε) and magnetic permeability (μ). The speed within the medium is then given by Equation 1.

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{Equation 1}$$

The effects of refraction are more easily analyzed through the use of the index of refraction (n). The index is defined as the ratio of the speed of propagation of EM radiation in a vacuum (*c*) to the phase velocity in the medium (*v*) as shown in Equation 2.

$$n = \frac{c}{v} \qquad \text{Equation 2}$$

The index of refraction will then always be a positive number greater than one. Propagation of a wave traveling with a phase velocity near the vacuum speed of light will have a value closer to one than a wave traveling with a lower phase velocity.

For EM waves travelling in the earth's atmosphere the index is generally so close to one that it is often easier to refer to the index in parts per million (ppm) rather than use the index directly. A scaled version of the index of refraction, called refractivity, often appears in error correction formulas.

Refractivity, designated as N, is related to the index of refraction by Equation 3.

$$N = (n - 1) \cdot 10^6 \qquad \text{Equation 3}$$

For example, the world-wide average refractivity at sea level is often given 316 N-units. Alternately, the index itself is expressed as 1.000316.

Refractivity is comprised of two parts: the contribution due to atmospheric pressure and the contribution due to the partial pressure of water vapor. Thus, refractivity will generally be lower in dry climates than in humid climates.

Refractivity decreases as the height above sea level increases, principally because the atmosphere thins as altitude increases. For example, refractivity at 4,000 feet above sea level is about 250-275 ppm.

Often the effects of refraction can be ignored at heights above 100,000 feet (30,480 meters) because the atmosphere is vanishingly thin at that height.

[2] Kraus, John, D. and Keith R. Carver. *Electromagnetics*, Second Edition. New York: McGraw-Hill Publishing, 1973. 366-367.

3. Snell's Law

Snell's Law can be used to determine the change in the direction of an EM wave as it crosses a boundary between dissimilar mediums. This is shown in Figure 2.

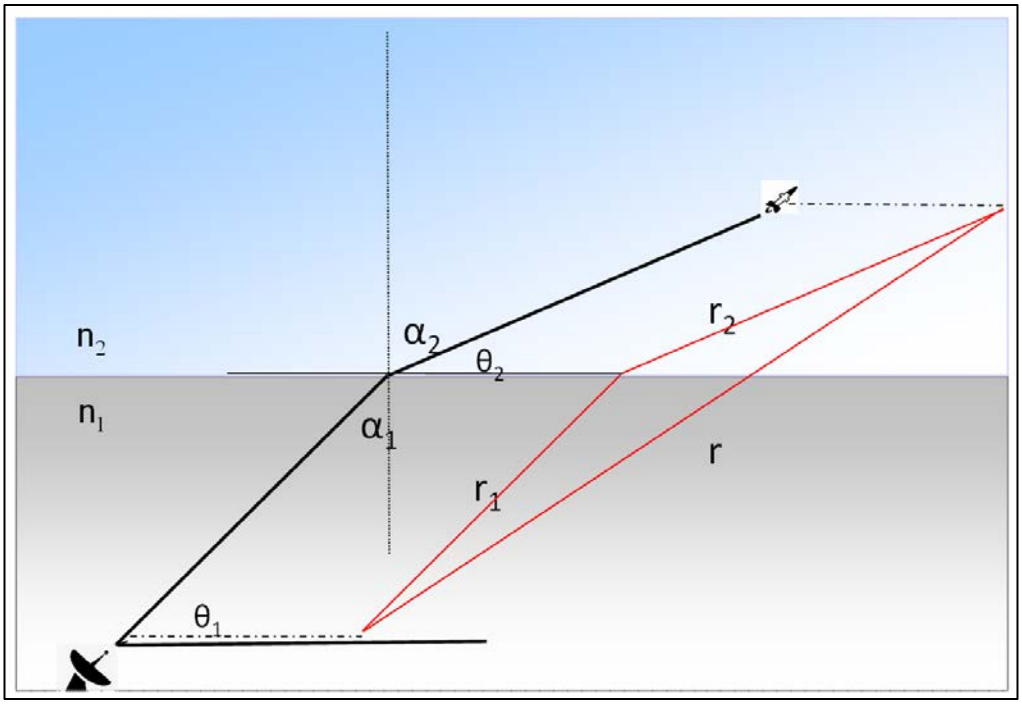

Figure 2. Snell's Law

The angular relationship in the path of the EM ray as it passes from one medium to another is given by Equation 4.

$$n_1 sin\alpha_1 = n_2 sin\alpha_2 \qquad \text{Equation 4}$$

Combining Snell's Law and the Law of Sines results in a form that provides a complete description of the ray path as it transits through discrete atmospheric layers from the radar to a distant object, as shown in Equation 5.

$$n_1 r_1 cos\theta_1 = n_2 r_2 cos\theta_2 \qquad \text{Equation 5}$$

This process can be extended to multiple atmospheric layers to provide better estimates of the true path length and change in the direction of propagation. Figure 3 illustrates the geometry for Snell's Law extended to multiple stratified layers. Reducing ΔN leads to a better estimate but requires an estimate of the index of refraction within each of the modelled atmospheric layers.

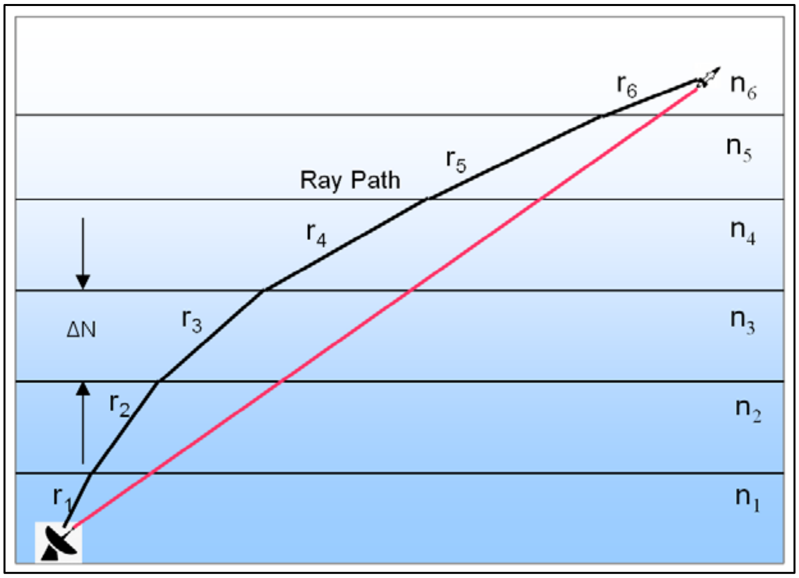

Figure 3. Stratified Atmospheric Layers

This simple Flat Earth model can be extended to the spherical model shown in Figure 4.

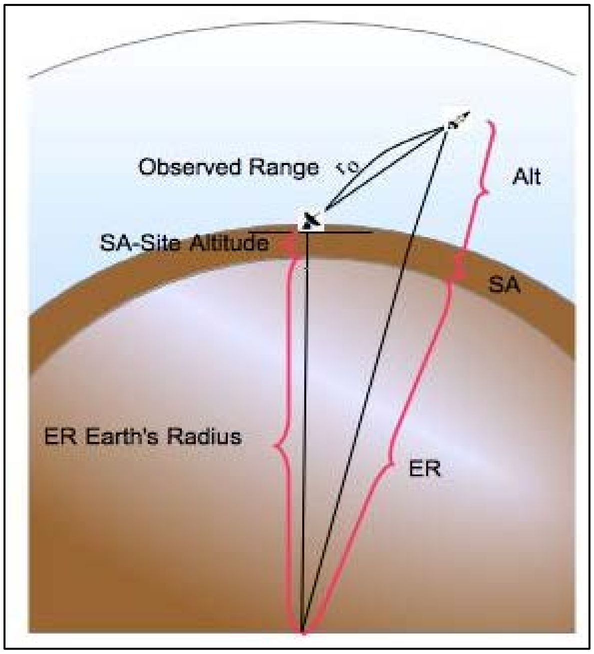

Figure 4. Spherical Earth Model

Here the atmospheric layers consist of spherical shells that surround the earth, extending from the surface to some sufficiently high altitude so that objects at heights above it will experience negligible refraction.

The geometry of the Spherical Earth model includes the earth's radius plus site altitude as one leg of a large triangle, the earth radius plus site altitude plus apparent target altitude as another leg, and the observed range is the third leg.

The ideal atmospheric model is a set of infinitely thin shells shaped by the gravitational field of the earth, each accurately reflecting the index of refraction at each point along the path.

4. Atmospheric Model

All refraction models require an estimate for the index of refraction for each layer. The earth's atmosphere is composed of a constantly changing medium resulting in different indices of refraction along the path of the EM wave. Bean and Thayer[3] developed a standard reference atmosphere using the measured surface refractivity and an exponential decay model to determine the approximate refractivity at any altitude above the surface, as shown in Equation 6.

$$N(h) = N_s e^{-k(h-h_0)} \qquad \text{Equation 6}$$

Where:

N(h) is the refractivity at height h;
Ns is the surface refractivity determined from surface weather measurements;
h0 is the surface height above sea level of the observing site;
k is a decay constant called the lapse rate.

Such a model is simplistic: it ignores lateral components of refraction, turbulence, temperature variations, and transient inversion layers; however, these systematic errors are often overshadowed by normal variations in the atmosphere, so they can often be safely ignored (Physical Science Laboratory, 1979). Corrections for refraction are usually limited to just changes in path length (range) and apparent height (elevation).

Model variations include differences in the lapse rate, limits on the height of the atmosphere, and step size between boundary layers. Typically, the lapse rate is determined by setting the top of the atmosphere at some height above which refraction is negligible. The lapse rate is then solved such that it produces the measured N_0 at the surface height and an assumed negligible index of refraction at the top of the atmosphere. More complex models may use multiple lapse rates to more accurately reflect the different meteorological layers within the troposphere.

5. Atmospheric Modeling Parameters

5.1 Earth Model

Refraction correction models use either a flat- or a curved-earth model. Flat-earth models are appropriate when the distance to a tracked object is such that the difference in the computed altitude for that object between a flat model and the curved model is negligible. The geometry for the Flat Earth model is simpler so it is often preferred if conditions justify its use.

[3] Bean, B. R. and G. D. Thayer. *CRPL Exponential Reference Atmosphere*. Washington: U.S. Dept. of Commerce, National Bureau of Standards, 1959.

5.2 Lapse Rate

The density of the atmosphere and hence its index of refraction at a given height is normally modeled as an exponential decay. There is a span of choices for the decay constant, called the lapse rate. Since the earth's atmosphere does not actually follow a precise exponential model, meteorologists often use different lapse rates for different altitude bands; however, a constant lapse rate generally provides a good estimate for the index when modeling the atmosphere for RF refraction errors.

One often-used method of determining a lapse rate is to create a decay constant that results in the index passing through the surface value and another point at an arbitrary selected "top" of the atmosphere. One often-used choice for the top of the atmosphere is 100,000 feet (30,480 meters) where the refractivity at that height is taken as 3.9 ppm. Figure 5 shows the exponential decay in refractivity from a starting point near sea level to 100,000 feet.

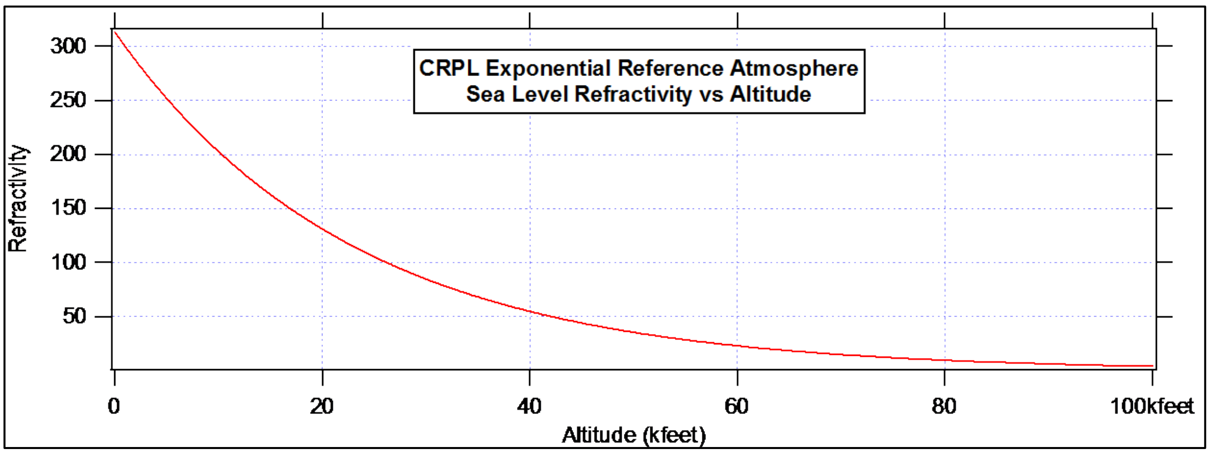

Figure 5. Central Radio Propagation Laboratory Exponential Reference Atmosphere

5.3 Surface Refractivity

Another choice is the starting value for the index of refraction (or equivalently the refractivity) at the surface of the earth. Standard models provide a value at sea level but many instrumentation radars are located above sea level. The estimate can be improved a little for higher elevations if the refractivity-observing site altitude is used instead of a sea-level value.

For example, the Central Radio Propagation Laboratory (CRPL) Exponential Atmospheric Reference Model (Bean and Thayer) gives the sea level refractivity as 313 ppm with a lapse rate of 0.143859km^{-1}; however, when the measured surface refractivity at a higher surface elevation is found to be 250 ppm, then the lapse rate is adjusted to 0.125625km^{-1}.

Other approaches model the temperature of the atmosphere exponentially and then compute refractivity through application of standard pressure, volume, and temperature formulas.

5.4 Stratified Boundary Layers

Atmospheric density changes continuously with height (as well as with other less-dominant factors). For computational reasons the atmosphere may be discretized into separate

layers, each with a constant refractivity value. The step size of the layer is an important choice. Large step sizes reduce the computational burden but they may not provide the correct level of modeling fidelity. Closely spaced boundaries add computational complexity and may not provide any additional accuracy over more widely spaced layers.

5.5 Water Vapor Contribution

The presence of water vapor in the atmosphere is an important factor for the refractive bending of an EM wave, especially if the frequency of the wave is in the RF band. Water vapor is measured in different ways and this leads to different equations. Often, they require different physical conversion factors. This results in many different methods for determining water vapor contribution to refractivity.

One standard technique developed by Smith and Weintraub[4] and used by the International Telemetry Union and others utilizes the local atmospheric temperature and the partial pressure of water vapor to compute the surface refractivity. This is shown in Equation 7.

$$N_s = N_{dry} + N_{wet} = \frac{77.6}{T}\left[P + 4810\frac{e}{T}\right] \qquad \text{Equation 7}$$

Where:

P is atmospheric pressure in millibars;
T is temperature in Kelvin;
e is the partial pressure of water vapor in the atmosphere.

Here the constant values were determined empirically by Smith and Weintraub based on measured dielectric constants for dry air and the thermodynamic properties of water vapor.

The partial pressure of water vapor represents a significant contribution to refractivity. For example, the refractivity for dry air at sea level is generally taken to be between 313 and 316 ppm; however, a test range in Hawaii uses a standard surface refractivity of 359 ppm, reflecting the higher absolute humidity of its subtropical climate.

6. Ray Tracing

We now have all the elements needed to fully solve the refraction problem. The exponential model of the atmosphere provides an estimate of the index of refraction at any height above an observer. The Spherical Earth model accounts for the curved shape of the atmosphere shaped by the earth's gravity. Snell's Law and the Law of Sines allows the geometry at each layer to be fully described.

We need only shrink the atmospheric layers to infinitesimal widths and then solve the resulting differential equations. Such a formulation is known as ray tracing because the solution traces the exact path of the EM wave as it propagates to a distant object.

Bean and Thayer demonstrated how Snell's Law can be adapted for infinitesimal changes in the index of refraction. Ray tracing begins by adding small delta values to the Snell's Law variables.

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[4] Smith, Ernest K. Jr. and Stanley Weintraub. "The Constants in the Equations for Atmospheric Refractive Index at Radio Frequencies." Journal of Research of the National Bureau of Standards, Vol 50, No. 1, January 1953.

Here Equation 5 is modified to include infinitesimal changes, as shown in Equation 8.

$$n_1 r_1 cos\theta_1 = (n_1 + \Delta n)(r_1 + \Delta r)cos(\theta_1 + \Delta\theta) \qquad \text{Equation 8}$$

Note, for this algorithm Bean and Thayer solved for the bending angle τ, which describes the downward curve of the ray at any point along its path. The bending angle is formed by the crossing of vectors created from the observed elevation angle and the tangent of the ray at the boundary crossing. The elevation correction is then found by applying simple planar geometry angle relations. A detailed diagram showing the relationship among the parameters used to develop the bending equations appears in Appendix A.

The limit as the deltas approach zero yields a differential equation. If the equation is expanded and the products of differentials are eliminated one obtains the classic expression for the bending of a radio wave, which is shown in Equation 9.

$$d\tau = -\frac{dn}{n} cot\theta \qquad \text{Equation 9}$$

When integrated over the path length, Equation 9 becomes Equation 10.

$$\tau_{[1,2]} = -\int_{n1}^{n2} \frac{dn}{n} cot\theta \qquad \text{Equation 10}$$

Where:

τ is the bending angle through which the ray has been bent from its original direction;
n is the index of refraction;
dn is the infinitesimal change in the index along the ray path;
θ is the observed elevation angle.

Using a similar approach, the distance along the longer curved ray path is then found with Equation 11.

$$R_{[e]} = \int_{r0}^{r} \frac{ndr}{sin\theta} \qquad \text{Equation 11}$$

The equations then provide for a direct solution of the path of the bent ray.

6.1 Ray Tracing Using Numerical Integration

An attractive procedure is to compute ray path angles by solving the integral equation version of Snell's Law; however, these integral equations must be solved using computationally intensive numerical techniques.

The BAE Systems[5] and NASA Ray Trace[6] routines use recursive numerical integration, iterating a fixed number of times or until the apparent height of the object reaches a stationary value. The NASA ray trace adds adaptive range corrections based on empirical measurements. A description of these two algorithms appears in Appendix B.

6.2 Simplifying Computational Procedures

Many of the refraction correction techniques used at the test ranges were developed at a time when the computational power to solve complex integral equations in a real-time environment did not exist. Because the integrals are computationally difficult to solve many refraction models rely on the simpler algorithms that provide approximate but adequate corrections for the effects of refraction.

A crude correction can be found using the 4/3 Earth Radius model. Here the refractive index is assumed to decrease linearly with height. The curvature of the rays is then such that, if plotted in a geometry where the earth has a radius *k* times greater than its true radius, they will appear as straight lines.[7] A standard value for *k* is 4/3. Such a model may provide reasonable results if the altitude of the object is relatively low. The 4/3 approximation is used only for "back of the envelope" guesswork. None of the test ranges use this method to correct for instrumentation radar errors.

Where conditions warrant, a simple Flat Earth model with fairly wide atmospheric layers may suffice. A spherical model must be used when the curvature of the earth becomes significant over the region of interest. Here the method is to compute exact values for a few specific points in a range-elevation volume of interest. These are then presented in a coverage plot. The correction factors can be interpolated by eye or the points can be fit to representative curves so that interpolations can be computed though mathematical algorithms.

Work by NASA in the mid-1960s for the Gemini Spacecraft program approximated the solution to the bending angle integral with a simple linear relationship to the surface refractivity, the cotangent of the observed elevation angle, and a polynomial with empirical coefficients.[8] This is shown in Equation 12.

Equation 12

$$\tau \approx [(N_s \cdot 10^6)(cot(\varepsilon))]\left[1.03585796 - \frac{1.072014 \cdot 10^{-2}}{\varepsilon} + \frac{1.279119 \cdot 10^{-8}}{\varepsilon^2} - \frac{1.227363 \cdot 10^{-8}}{\varepsilon^3}\right]$$

Here ε is the observed elevation angle. The correction was found to be valid for observed elevations between 2 and 10 degrees. Above 10 degrees only the first term in brackets is used.

Models often compute a few salient points over the region and then fit curves through the points to provide a simple method to interpolate between them. Often, empirical corrections are

---

[5] BAE System, "RF Refraction Correction," n.d. Available on request to the RCC Secretariat.
[6] Agee, B. and Don Sammon. "NASA Ray Trace Algorithm," adapted from unknown NASA author. Unpublished C Source Code, September 7, 1995. Available on request to the RCC Secretariat.
[7] Merrill I. Skolnik. *Radar Handbook*. New York: McGraw-Hill, 1970, 2-45.
[8] P. E. Schmid. *Atmospheric Tracking Errors at S- and C-Band Frequencies*. NASA Technical Note TN D-3470. Washington, D.C: National Aeronautics and Space Administration, August 1966, p. 9.

used to improve the initial approximation. The simple Kermit-Pearson formulation displayed in Equation 13, appears to be based on this approach.

$$\Delta e = \frac{k1e \cdot r \cdot cos(el)}{k2e + r \cdot sin(el)} \qquad \text{Equation 13}$$

One method by Maron[9] computes accurate correction values over a range-elevation grid and then fits spline curves through the subsets of the computed points. The correction for a given range-elevation observation pair is then found by interpolation between smooth mathematical curves.

Another approach by Milnarich[10] is to recast the non-integral form of Snell's Law so that it is suitable for an iterative solution. The iteration then computes, in reverse fashion, an observation angle error using an exponential atmosphere model for the index of refraction. It stops when the sum of the geometric and error correction matches the actual observed angle.

Variations of these simplifying techniques lead to the plethora of different refraction correction algorithms used at the test ranges.

Preliminary comparisons of the various approaches suggest that both ray tracing and curve fitting approaches can provide reasonably accurate results. Comparisons at White Sands Missile Range (WSMR) between the simple Kermit-Pearson Flat Earth model and the sophisticated NASA Ray trace algorithm suggest that, at least at elevations above five degrees, the difference between methods may be on the same order as the differences caused by small variations in the atmosphere. The general rule appears to be that applying some correction for refraction is more important than the choice of the method used to implement the correction.

6.3 Other Computational Factors

Even when refraction corrections are applied to radar track data, the test ranges may use different techniques to implement the corrections. For example, some algorithms do not attempt to estimate refraction when the elevation is below zero. Others mirror the below-zero correction with its positive angle counterpart to avoid any discontinuities.

Some algorithms add empirical corrections, especially for low elevation angles where the models tend to deviate markedly from the actual EM propagation.

In addition to applying refraction to outgoing data, some ranges apply a reverse refraction to incoming cueing data so that an instrument will be cued to the observed rather than actual location of an object.

Other procedures call for applying the correction only under certain conditions. For example, radar data may not be corrected for refraction unless the radar is indicating a close-loop track of an object.

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[9] D. E. Maron. "Refraction Correction Algorithm," MOTR Program Memorandum MOTRPM-2000-020. RCA Corp, June 3, 1985. Available on request to the RCC Secretariat.

[10] Paul Milnarich. "A Method for Predicting the Apparent Elevation Angle for a Tropospheric Object From a Specified Elevation Angle and Height" (dissertation, UTEP, 1966), http://digitalcommons.utep.edu/dissertations/AAIEP00334/.

7. Survey of Range Radar Refraction Correction Methods

Appendix C[11] of this document describes the refraction correction methods, and to the extent that they are available, the model parameters, at each of the participating test ranges.

8. Future Work

The long-term goal is to select a standard algorithm that is suitable for a wide range of conditions and that can be used at most member ranges to correct radar refraction errors. Future work should first include a down selection of suitable candidate algorithms, coding the candidate algorithms in a common software language such as MATLAB, and then evaluating their effectiveness on multiple sets of data culled from member ranges.

One candidate algorithm from BAE Systems, the RIR980 refraction model, has already been deployed on several test ranges. Its methods are well documented and code to implement the algorithm is readily available. As such, it is a good candidate for a refraction correction standard.

A slightly more sophisticated algorithm, the NASA Ray Trace, uses a similar but higher-order integration technique and it includes an iterative loop and empirical adjustments for very-low-elevation angle observations. A description of these algorithms with code snippets appears in Appendix B.

[11] Range Commanders Council. "Member Range Refraction Corrections." 266-16 Appendix C. November 2016. Available to RCC members with Private Portal access at https://wsdmext.wsmr.army.mil/site/rccpri/Publications/266-16_Survey_Radar_Refraction_Error_Corrections/266-16_Appendix_C.pdf.

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APPENDIX A

Bending Angle Geometry

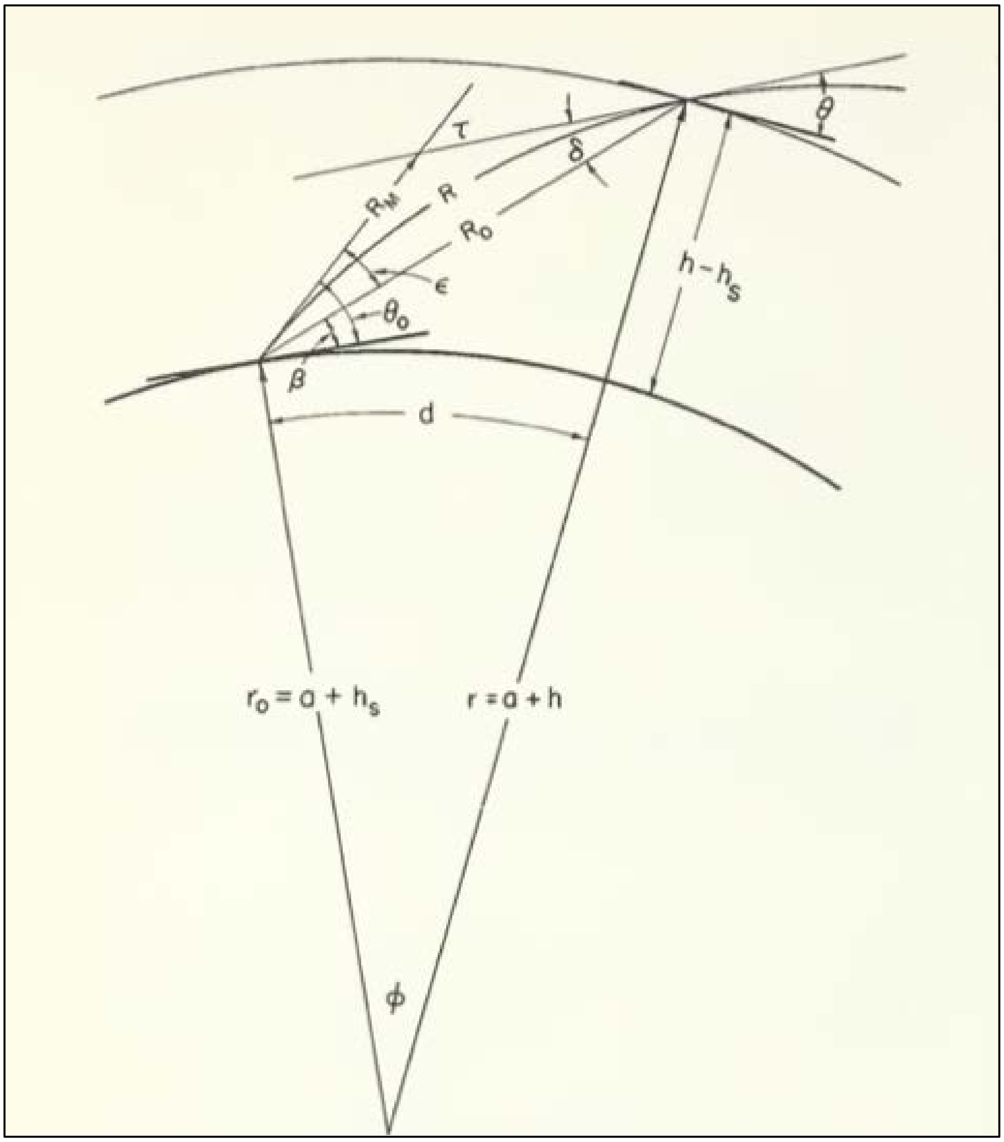

Figure A-1. Geometry Used for Finding the Bending Angle τ

r_o Earth radius
h_s observation site elevation
h target altitude
θ_o observed elevation angle
ε elevation refraction error
β true elevation angle
R_m measured range
R ray path
R_0 true range

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APPENDIX B

Ray Tracing Techniques

B.1 BAE and NASA Ray Trace Algorithms

Ranges having FPS-16 upgrades developed by BAE Systems of Fort Walton Beach FL use identical refraction correction algorithms (BAE Systems). Currently Yuma's FPS-16 class radars and Eglin's RIR980s utilize this algorithm. Another range, WSMR, is slated to begin using this algorithm in one upgraded FPS-16 radar as soon as it becomes operational in January 2017. Four other upgrades are planned in the near future and all of them will use the BAE refraction correction algorithm.

Eglin reports this algorithm provides results that are consistent with older algorithms developed for their FPS-16 radars. If other ranges can confirm its effectiveness then it could be the basis of an RCC Refraction Correction standard.

The WSMR radar community also uses a NASA Ray trace routine that is similar to the BAE formulation. The exact origins of this routine have been lost but Bill Agee, a mathematician at WSMR, modified the original NASA Fortran code to include empirically derived range bending for targets at long ranges and low elevation angles. Don Sammon, also at WSMR, later converted the Fortran code to C.

Both the BAE and NASA Ray trace routines use similar approaches to recursively solve the bending angle integral using a Gaussian Quadrature numerical technique. The BAE algorithm uses a fourth-order polynomial while the NASA routine uses an eighth-order polynomial.

The differential equation for the bending angle is shown in Equation 9. Integrating over the path length yields Equation 10.

This integral must be solved numerically. One could compute function values at equally spaced points and then multiply each result by a set of weighting coefficients. The familiar Trapezoid Rule is one such technique; however, these ray trace routines use a more sophisticated Gaussian Quadrature technique to solve the integral.

Gaussian Quadrature methods provide the additional degrees of freedom by selecting not only the weighting coefficients, but also the location of the abscissas.[12] Gaussian Quadrature methods work only if the integrand is very smooth and free of discontinuities.

One attractive feature of the Gaussian Quadrature is that, when the integrand is arranged as a polynomial times some weighting function, and the weights and abscissas are carefully selected, the method can provide exact results. Hence the approximation shown in Equation B-1 can be exact if f(x) is a polynomial.

$$\int_a^b W(x) f(x) dx \approx \sum_{i=1}^{N} w_i f(x_i) \qquad \text{Equation B-1}$$

[12] William H. Press, et. al. "Gaussian Quadrature and Orthogonal Polynomials" in *Numerical Recipes in C*. Cambridge University Press, New York 1988. 131-137.

The selection of the abscissas at which the integral is to be evaluated corresponds to the roots of the polynomial. The weighting function is tied to the theory of orthogonal polynomials. The determination of the orthogonal weighting function exceeds the scope of this document; however, numerical values are tabulated in many mathematical handbooks.

Both routines solve the integral using the Gaussian Quadrature procedure and then iterate several times to make sure that the result is stable. The NASA Ray Trace also is adaptive in the sense that it adds empirical range bending when the observed range is greater than 50,000 yards and the observed elevation is less than 3 degrees.

B.2 NASA Ray Trace Algorithm

The C code for the NASA Ray Trace program (Agee and Sammon, 1995), with additional comments, is included below.

```
//-----------------------------------------------------------
// NASA Ray Trace algorithm
//-----------------------------------------------------------
/*
 This routine accepts apparent elevation and range, and
 computes a negative delta e and delta r correction
 using a 9 point gaussian quadrature integrator.

 The algorithm iterates the integral solution up to five times
 or until the change in the computed target altitude is less
 than one foot between successive interations.

 The algorithm also includes an emperically derived bending
 correction for range when measured range is greater than
 50kyards and the elevation is <3 deg.

  I/O Parameters:
   rad = earth radius in feet (at station)
   eo  = apparent elevation in radians
   ro  = apparent range in yards
   fn  = surface index of refraction (e.g.  1.000275)
   de  = elevation correction in radians
   dr  = range correction in feet

   Algorithm is based on Bill Agee's (NRO-AM) adaptation of the NASA ray
   trace program with empirically derived range bending corrections
   for elevations below 3 degrees.

   Don Sammon  NRO-DR-S  (575)-678-2950  7 Sep 95
   Additional comments added 2 June 2016

*/

#define pi2 1.570796327
raytr(rad,ro,eo,fn,de,dr)
  double  rad;
  double  ro;
  double  eo;
  double  fn;
  double  *de;
  double  *dr;
```

```
{

  //--------------------------------------------------------------------
  //declarations
  //--------------------------------------------------------------------
  //   constants weighting values for gaussian integration
  double  a1[9]={.04063719418,.09032408035,.1303053482,
                 .15617353850,.16511967750,.1561735385,
                 .13030534820,.09032408035,.04063719418
                  };

  double  u[9]={-.0159198802,-.0819844463,-.1933142836,
                -.3378732883,-.5000000000,-.6621267117,
                -.8066857164,-.9180155537,-.9840801198
                 };

  // empircally derived coefficients for the range bending correction
  double  rpl[14]={ 0.5,0.7,1.0,1.2,1.4,1.6,1.8,
                    2.0,2.5,3.0,4.0,7.0,10.,40.0
                    };
  double  po[14]= {-.7500,-.3028,.1424,.3104,.4246,.5058,.5661,
                    .6127,  .6929,.7415,.7991,.8681,.8944,.9387
                  };

  double  p1[14]= {   0.,.4134,.4997,.5330,.5531,.5648,.5714,
                   .5753,.5811,.5865,.5929,.6017,.6051,.6111
                  };

  double  p2[14]= {    0.,.01993,.03836,.05136,.06346,.07394,.08257,
                   .08943,.10035,.10591,.11160,.11698,.11871,.12128
                  };

  double  p3[14]= { .47,0.74,1.15,1.36,1.53,1.66,1.76,
                   1.85,2.00,2.10,2.22,2.38,2.45,2.56
                    };

  double  cse,rat,sth;
  double  fnm1,stk;
  double  stn;
  double  a,dn,y2,y1,fnj,hj,c1,c2,c3,temp;
  double  rp,ep;
  double  rr,poc,p1c,p2c,p3c,bend;
  double  ste,theta,et;
  double  sthp,dh;
  double  lde,ldr;  //local copies of de and dr
  int     i,j,ib;

  //  --------------------------------------------------------------------
  //  entry point intialization and parameter boundary checking
  //  --------------------------------------------------------------------
  //  boundary checks: return if range is less than 0,if the elevation is
  //  less than zero or greater than 90 degrees, or index is < 0.
  ldr=0.;
  lde=0.;
  *dr = ldr;
  *de = lde;
  if (eo<  0)return(1);
  if (eo>pi2)return(1);
  if (ro<=0 )return(1);
```

```
if ((fn -1.)<0)return(1);

// compute apparent height of target above the surface of the geoid
// using pythagorus on the triangle formed by the earth radaius+site
// altitude as one leg and observed range and elevation as the second leg.
// apparent height is then found by subtracting the earth radius from the
// computed third leg
rat = ro/rad;
sth = rad*(sqrt(1.+2.*rat*sin(eo)+rat*rat)-1.);
if (sth<0.1)return(1); // exit if target altitude is neglibly small

// compute the lapse rate(exponential decay constant) from the surface
// refractivity provided as an input parameter
// Surface Refractivity is computed in a separate procedure using wet bulb,
// dry bulb, and pressure measurements taken at the radar site prior
// to the mission.
// The other value used to compute the laspe rate is an assumed N=3.36
// at 100,000 feet altitude.
// The Refractivity at any altitude is then N=Ns*exp(-kh) where k is the
// laspe rate consistent with the computed Ns and N(100kft)=3.36

fnm1 = fn - 1.0;
stk = log(fnm1/.00000336)/100000.;

//-----------------------------------------------------------------------
// iteration loop
// The algorithm performs up to five iterations of the integral
// ray trace equations or until the apparent heght of the target changes
// less than one foot between successive iterations
//-----------------------------------------------------------------------
cse = cos(eo);
for(j=0;j<5;j++){
  // for apparent target heights up to 1 million feet use the laspe rate
  // found above to estimate the index of refraction at that height
  // For heights over 1 million feet index=1
  if (sth<1e6) stn = 1.0 + fnm1 * exp(-sth * stk);
   else stn = 1.0;

  //-----------------------------------------------
  //  Solve the Ray Trace integral equation using a
  //  9th order Gaussian quadrature integration
  //  see Numerical Recipes in C Ch 4.5 pp 131-137
  a = 1.0 - stn/fn;
  dn = fn - stn;
  y2 = 0.0;
  y1 = y2;

  for(i=0;i<9;i++){
    fnj  = dn * u[i] + fnm1;
    hj   = log(fnm1/fnj)/stk;
    c1   = 1.0 + a * u[i];
    c2   = 1.0 + hj/rad;
    temp = a1[i]/(fn * sqrt(c1*c1*c2*c2-cse*cse));
    y1   = y1 + c1 * c2 * temp;
    y2   = y2 + temp/c1;
  }
  // compute the true elevation angle and delta e
  c3 = 1.0 + sth/rad;
  ste = acos(fn * cse/(stn*c3));
```

```
    theta = ste-eo+fn*cse*a*y2;
    et = atan((cos(theta) - 1.0/c3)/sin(theta));
    if (eo>1.57077887) et = eo;
    lde = et - eo;

   // compute the delta range and alpha angle
    ldr = -a * y1 * fn * fn/stk;

  //---------------------------------------------------
  //if el<3 degrees and range > 50kyards, setup empirical range bending
    if ((eo<0.05235987756)&&(ro>500000.)){
      rp = ro * 1.e-6;
      if (rp > 40.0) rp = 40.0;
      ep = 3.  - eo*57.2957795131;

      // find the correct range index to select
      // the correct coefficnets for the adapative interpolation
        for(ib=1;ib<14;ib++){
          if (rp <= rpl[ib])break;
        }

      // interpolate the bending using the selected co-efficients
      rr = (rp - rpl[ib-1])/(rpl[ib] - rpl[ib-1]);
      poc = po[ib-1] + rr * (po[ib] - po[ib-1]);
      p1c = p1[ib-1] + rr * (p1[ib] - p1[ib-1]);
      p2c = p2[ib-1] + rr * (p2[ib] - p2[ib-1]);
      p3c = p3[ib-1] + rr * (p3[ib] - p3[ib-1]);

      // compute bending effect from the co-efficients
      bend = p3c - exp(poc + p1c * ep + p2c * ep * ep);
      bend = bend * fnm1/.00036;
      ldr = ldr + bend;
    }

    //---------------------------------------------------
    // compute a new apparent height above the geoid
    rat = (ro + ldr)/rad;
    sthp = rad * (sqrt(1.0 + 2.0*rat*sin(et)+rat*rat) -1.0);

    //---------------------------------------------------
    // check for apparent height convergence
    dh = fabs(sth - sthp);
    sth = sthp;
    if (dh<1.)break;
  }
  //return the results
  *dr=ldr;
  *de=lde;
  return(0);
}
```

APPENDIX C

Member Range Refraction Corrections

Appendix C is available to RCC members with Private Page access at https://wsdmext.wsmr.army.mil/site/rccpri/Publications/266-16_Survey_Radar_Refraction_Error_Corrections/266-16_Appendix_C.pdf. It is also available to US Government agencies and their contractors from the RCC Secretariat at (575) 678-1107 or usarmy.wsmr.atec.list.rcc@mail.mil.

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APPENDIX D

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